6/H–29 (viii) (b) (Syllabus–2015)

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(April)

MATHEMATICS

(Honours)

(Operation Research)

(HOPT-62 : OP2)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Explain the following terms : 2+3+2=7

   (i) Convex set

   (ii) General linear programming problem

   (iii) Optimum solution to a general LPP

(Turn Over)

(b) The manager of an oil refinery must decide on the optimal mix of two possible blending processes of which the inputs and outputs per production run are as follows :

Process	Input		Output	
	Crude A	Crude B	Gasoline X	Gasoline Y
1	5	3	5	8
2	4	5	4	4

The maximum amounts available of crudes A and B are 200 units and 150 units respectively. Market requirements show that at least 100 units of gasoline X and 80 units of gasoline Y must be produced. The profits per production run from process 1 and process 2 are ₹ 300 and ₹ 400 respectively. Formulate the problem as an LPP.

2. (a) An electronic company manufactures two radio models each on a separate production line. The daily capacity of the first line is 60 radios and that of the second line is 75 radios. Each unit of the first model uses 10 pieces of a certain electronic component whereas each unit of the second model requires

(Continued)

8 pieces of the same component. The maximum daily availability of the special component is 800 pieces. The profit per unit of the first and second models are ₹ 500 and ₹ 400 respectively. Formulate the problem as an LPP model and determine graphically the optimal daily production of each radio model. 8

(b) Solve the following LPP graphically : 7

Maximize $Z = x + y$

subject to the constraints

$$x + y \leq 1$$
$$-3x + y \geq 3$$
$$x \geq 0,\ y \geq 0$$

UNIT—II

3. (a) Explain the following terms : 3+3+3=9

(i) Canonical and standard form of an LPP

(ii) Slack and surplus variable in a general LPP

(iii) Primal and dual problems in an LPP

(Turn Over)

(b) Obtain the dual problem of the following problem :

$$\text{Minimize } Z = x_1 - 3x_2 - 2x_3$$

subject to the constraints

$$3x_1 - x_2 + 2x_3 \le 7$$
$$2x_1 - 4x_2 \ge 12$$
$$-4x_1 + 3x_2 + 8x_3 = 10$$
$$x_1, x_2 \ge 0$$

and x_3 is unrestricted.

4. (a) Show that the dual of the dual problem is the primal.

(b) Write down the algorithm of the simplex method to solve an LPP.

UNIT—III

5. (a) Use simplex method to solve the following LPP :

$$\text{Maximize } Z = 4x_1 + 10x_2$$

subject to the constraints

$$2x_1 + x_2 \le 50$$
$$2x_1 + 5x_2 \le 100$$
$$2x_1 + 3x_2 \le 90$$
$$x_1 \ge 0 \text{ and } x_2 \ge 0$$

(Continued

(b) Determine whether the following two-person zero-sum game is strictly determinable and fair. If it is so, give the optimum strategy for each player : 5

Player B

$$\text{Player } A \begin{bmatrix} -5 & 2 \\ -7 & -4 \end{bmatrix}$$

6. (a) Explain maximin-minimax principle with an example. 5

(b) Determine the range of value of p and q that will make the pay-off element a_{22} a saddle point for the game whose pay-off matrix is given below : 5

Player B

$$\text{Player } A \begin{bmatrix} 2 & 4 & 7 \\ 10 & 7 & q \\ 4 & p & 8 \end{bmatrix}$$

(c) For the game with pay-off matrix

Player B

$$\text{Player } A \begin{bmatrix} 10 & 5 & -2 \\ 6 & 7 & 3 \\ 4 & 8 & 4 \end{bmatrix}$$

determine the best strategies for players A and B and also the values of the game for them. Is this game (i) fair and (ii) strictly determinable? 5

(Turn Over)

UNIT—IV

7. (a) Reduce the following pay-off matrix to a 2×2 matrix by dominance property and then solve the problem :

Player B

$$\text{Player } A \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 0 & 2 & 1 \\ 4 & 3 & 1 & 3 & 2 \\ 4 & 3 & 4 & -1 & 2 \end{bmatrix}$$

(b) Solve the following game :

Player B

$$\text{Player } A \begin{bmatrix} 4 & 2 & 4 \\ 2 & 4 & 0 \\ 4 & 0 & 8 \end{bmatrix}$$

8. (a) Solve the following game by linear programming technique :

Player B

$$\text{Player } A \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{matrix} B_1 & B_2 & B_3 \end{matrix} \\ \begin{bmatrix} 9 & 1 & 4 \\ 0 & 6 & 3 \\ 5 & 2 & 8 \end{bmatrix} \end{array}$$

(b) Solve the following 2×3 game graphically :

Player B

$$\text{Player } A \begin{bmatrix} 1 & 3 & 11 \\ 8 & 5 & 2 \end{bmatrix}$$

(Continued)

UNIT—V

9. (a) Explain the following terms : 2½+2½=5

(i) Stochastic matrices and regular stochastic matrices

(ii) Brand switching analysis

(b) There are 2 white marbles in urn A and 3 red marbles in urn B. At each step of the process a marble is selected from each urn and the two marbles selected are interchanged. Let the state a_i of the system be the number i of red marbles in urn A.

(i) Find the transition matrix P.

(ii) What is the probability that there are 2 red marbles in urn A after 3 steps?

(iii) In the long run, what is the probability that there are 2 red marbles in urn A? 10

10. (a) Explain the following terms : 2½+2½=5

(i) Fixed points of square matrices

(ii) Absorbing states

(Turn Over)

(b) Let P be the transition matrix of a Markov chain. Then prove that the n-step transition matrix is equal to the nth power of P, i.e., $P^{(n)} = P^n$.

(c) A man either drives his car or takes a train to work each day. Suppose he never takes the train two days in a row; but if he drives to work, then the next day he is just as likely to drive again or take a train. What is the probability that he will change the state from driving to taking the train after 4 days?

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